5 Beams and Columns

Newton's Laws

During the academic years 1665 and 1666, Isaac Newton, then in his early twenties, was prevented from attending Cambridge University by the spread of the plague throughout England. This was a blessing in disguise for him and a boon for humanity. Cloistered in his mother's home at Woolsthorpe, Newton was able to ponder the deep questions of physics he had set his mind to answer, and to develop the ideas of his gravitational theory. These ideas were so daring for the time as to approach absurdity. He postulated, for example, that bodies could exert forces at a distance without material contact, contrary to all physical evidence. Even so, his law of gravitation predicted exactly why the moon rotated about the earth in an elliptic trajectory in almost exactly 27 and 1/3 days and the earth around the sun in another elliptic trajectory in 365 days, 5 hours, 48 minutes and 46 seconds. It also "explained" why the legendary apple fell on his head. Not for 250 years would a genius of Newton's magnitude, Einstein, come up with another even more daring and abstract assumption, denying the existence of "action at a distance" and improving on the accuracy of Newton's results.

Newton, with the modesty of some great geniuses, never asserted to have explained why bodies attract each other. He simply stated that bodies behave *as if* they attracted each other with a force governed by his gravitational law. But, having at least assumed a cause of motion, he

proceeded to describe how motion takes place. His three laws of motion allow the determination of the speed with which all bodies move, from the apple to the sun's planets. His first and third laws, when added to that of elasticity, are sufficient to solve almost all structural problems. His synthesis remains possibly the greatest in the history of science.

The first law of motion, as applied to structures, states that a body at rest will not move unless a new, *unbalanced* force is applied to it. The third states that, when a body is at rest, for each force applied to it there corresponds an equal and opposite balancing *reaction*, also applied to it. Since we want our structures *not* to move, except for the miniscule displacements due to their elasticity, Newton's laws of rest are the fundamental laws ruling the balance that must exist between all the forces applied to a structure.

In physics a body at rest is said to be in *equilibrium*, from the identical Latin word which means "equal weights" or balance. An understanding of two particularly simple aspects of the laws of equilibrium is essential to an insight into how structures work.

Translational Equilibrium

Consider an elevator hanging from its cables at a given floor. The pull of gravity on its cabin and occupants, that is, their total weight, acts downward. As the elevator does not move (is in equilibrium), a force upward must be exerted on it equal in magnitude to its total weight. This force can only be exerted by the cables. We conclude that the cables pull up on the elevator with such a force. The elevator is thus acted upon by two equal and opposite vertical forces and is in equilibrium in the *vertical* direction. According to Newton's third law, if we call action the force due to the weight, the pull of the cables constitutes the equal and opposite reaction. Similarly, if we stand on the floor, the pull of the earth (gravity) exerts a force pulling down on our body and, since we do not move down, an equal and opposite force must be exerted on it. This can only come from the floor pushing up on our feet, thus exerting a reaction (up) equal and opposite to the weight (down) of our body. Our body is in *vertical equilibrium.*

Consider now two groups of children pulling on the two ends of a rope. If neither group prevails, the pull of one group must be equal to that of the other and the rope, under the action of these two equal and opposite forces, is in equilibrium in the *horizontal* direction. If one group prevails, their force is greater than that of the opposing group and equilibrium is lost; the rope moves; it is not in equilibrium any more. In the first case, it may be hard to decide which of the two equal and opposite pulls is the action and which is the reaction. It is really a question of semantics and, perhaps, if one's boy or girl belongs to one group, on purely psychological grounds, we might be inclined to call this group's pull the action and the other's the reaction.

Identical considerations govern the equilibrium of structures. Since the total weight of one of the World Trade Center towers is approximately 140,000 tons and acts down, we know that the soil must exert on the foundation of the tower a force up of 140,000 tons. The soil under the tower must be quite hard to develop such a reaction. And so it is on Manhattan, an island which consists mostly of very solid rock. Similarly, if a wind exerts a horizontal pressure of, say, 30 pounds on each square foot of one of the tower's faces, which measure about 175,000 square feet, the total wind force on that face equals 30 times 175,000 square feet, or over 5,000 tons and the soil under the building must also react *horizontally*, in a direction opposite to that of the wind, with a force of 5,000 tons. If the soil were unable to do so, the tower would slide in the direction of the wind.

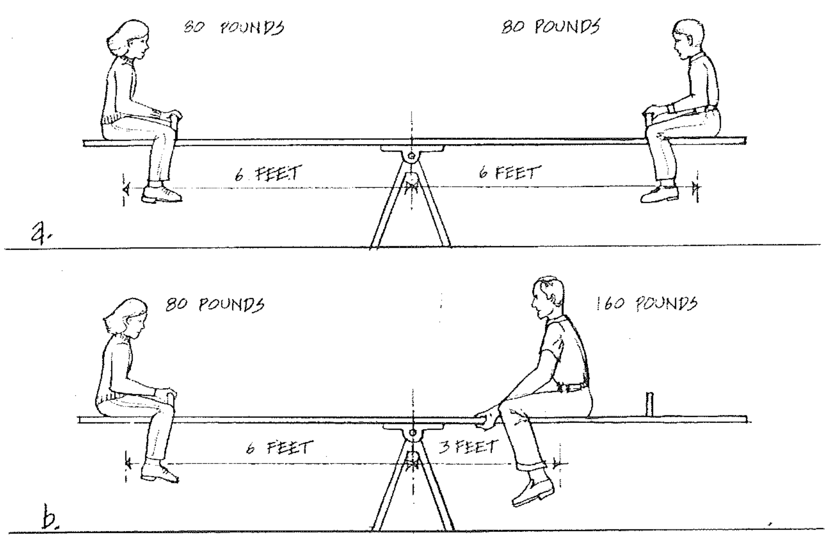
The two basic actions of tension and compression are usually developed in structural elements in equilibrium by equal and opposite forces acting along the center line or *axis* of the element. A steel column, of a weight usually negligible in comparison with the load it supports, is set in compression by the action down of the load's weight and the reaction up of its footing. One may say that the weight flows down the column until it is stopped by the footing. Similarly, the weight of the elevator pulls on its cables and we can visualize this pull travelling up along the cables until it is stopped at the top by the pulleys. One can "feel" axial tension or compression by pulling or pushing on a doorhandle. The pull or push exerted by our body and balanced by the doorknob's reaction is felt by the muscles of our arm along which the pull or push travels.

From the Latin word *translare,* meaning to move in the same direction, equilibrium in a given direction is called *translational equilibrium.* For a body to be at rest it must move neither vertically nor horizontally. For example, a building will be at rest in total translational equilibrium if it does not move vertically or in either of the two directions parallel to its faces. Total translation equilibrium is thus satisfied by three conditions of translational equilibrium: vertical, parallel to one face, and parallel to the other face.

Rotational Equilibrium

If an empty cereal box is placed in front of an electric fan, it will move under the action of the air flow. If it rests on one of its long narrow sides, it will probably move horizontally, but if it rests on one of its short, narrow sides (its bottom), it will no doubt topple over before it moves horizontally. The cereal box is a good model for a tall building not well anchored into the ground and acted upon by a strong wind. Although in the second case the box was in vertical and horizontal equilibrium, since it did not slide or move up or down, it was not at rest since it toppled over. This motion occurs when the box turns around its lower edge on the side opposite to the wind or, as we say in physics, *rotates* around this leeward edge. To maintain the box at rest this rotational motion must also be prevented.

To understand the requirements of *rotational equilibrium* consider a see-saw with two children of identical weight sitting on opposite ends (Fig. 5.1a). The see-saw does not move vertically because the reaction up of its pivot balances the weights down of the two children (it is equal to twice the weight of one child). The see-saw does not rotate either, unless



5.1 ROTATIONAL EQUILIBRIUM OF SEE-SAW

one of the children moves nearer the pivot, in which case the see-saw starts rotating down on the side of the other child and is not in rotational equilibrium anymore. Clearly rotational equilibrium requires not only that the weights of the two children be equal, but that their distances from the pivot also be equal. Let now a father weighing *twice* as much as his child sit on the see-saw and try to balance it (Fig. 5.1b). He succeeds if he sits *half* the distance of the child from the pivot. The two weights on opposite sides of the see-saw's pivot do not have to be equal for rotational equilibrium. What must be equal is the weight times its distance from the pivot. This is nothing else but the well-known law of the lever, which states that rotational equilibrium requires *equal products of forces times lever arms* (as distances from the pivot are called) tending to make the body rotate in opposite directions.

If one now fills the upright cereal box with enough sand or stones and blows on it again with a fan, the box will not topple over; it will be in rotational equilibrium. This occurs whenever the wind force times its vertical lever arm from the leeward edge is less than or at most equal to the weight of the building times its horizotal lever arm from the same edge (Fig. 5.2).* It is thus seen that the rotational action of a force can be balanced by that of another force in a different direction.

Simple as this may seem, the task of the structure is to guarantee translational and rotational equilibrium of the building under the action of any and all forces and reactions applied to it, including, of course, its own weight. The task of the engineer is to shape and dimension the chosen structural materials so that the structure may produce equilibrium without breaking up, and with acceptably small elastic displacements.

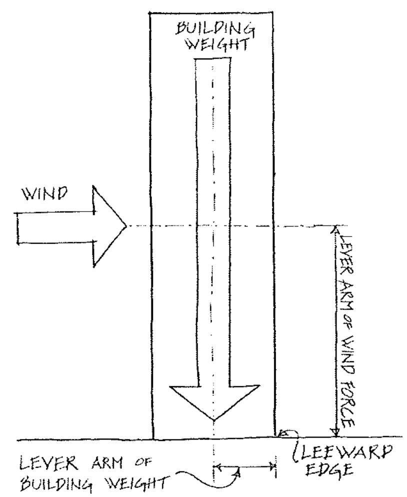
Beam Action

Equilibrium guarantees the stability of an entire building and, of course, of each one of its parts. In the California earthquake of 1971 a small hospital building toppled over without suffering serious damage. Its strength was sufficient, but not its equilibrium capacity. To prevent the toppling of a very high building, when the building's weight is not sufficient to counteract the wind, its columns are anchored into a founda-

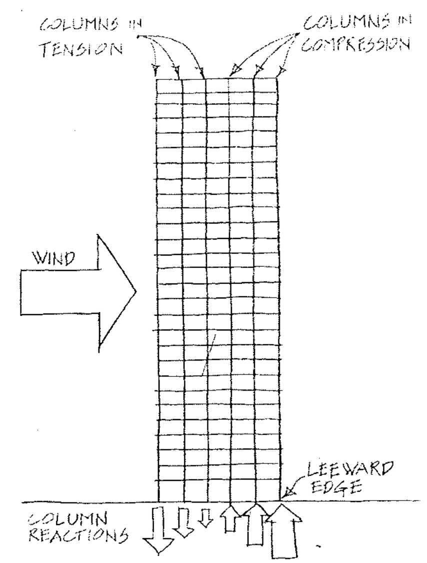
* For example, in one of the World Trade Center towers the horizontal wind force of 5,000 tons has a vertical lever arm of 700 feet (half the tower height) and a product equal to 5,000 × 700 = 3,500,000 or 3.5 million ton-feet. The tower weight of about 140,000 tons has a horizontal lever arm equal to 60 feet (half the tower width) and a product equal to 140,000 × 60 = 8,400,000 or 8.4 million ton-feet. The weight wins and the tower does not topple over.

tion deep in the ground. It this case the building bends slightly under wind pressure, but remains in rotational equilibrium in part due to the counteraction of its weight but also due to the forces exerted by the anchored columns. The windward columns are set in tension and those on the leeward side in compression (Fig. 5.3), thus creating a tendency to turn the building against the wind. The wind drift or lateral displacement of the top of the building is due almost entirely to the lengthening of the windward columns under tension and the shortening of the leeward columns under compression. Since the floors are rigidly connected to the columns, they tilt slightly and remain at right angles to them. All these deformations are so minute that they cannot be detected by the naked eye, but give to the building a slightly deflected curved shape.

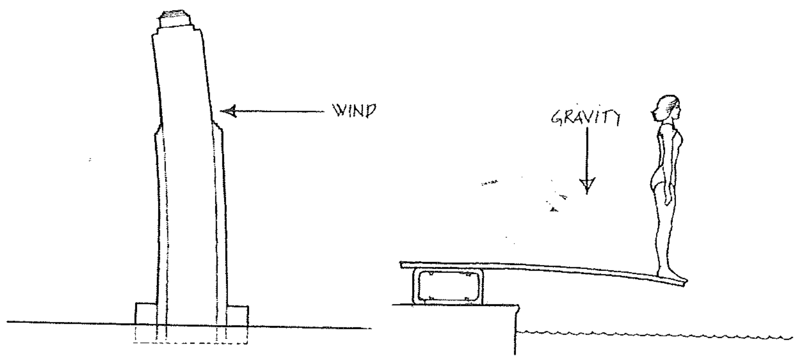
The shortening of a column under the action of an allowable compressive axial force or the lengthening of a cable under an allowable tensile force is extremely small. In the building under lateral wind load, we find, however, that the *lateral* deflections may be substantial. The columns of a 1,000-foot skyscraper shorten under the vertical loads by only one tenth of an inch, but the top of the skyscraper may bend out several feet under the action of lateral loads. The deflections due to loads perpendicular to



5.2 ROTATIONAL EQUILIBRIUM OF HIGH-RISE BUILDING UNDER WIND



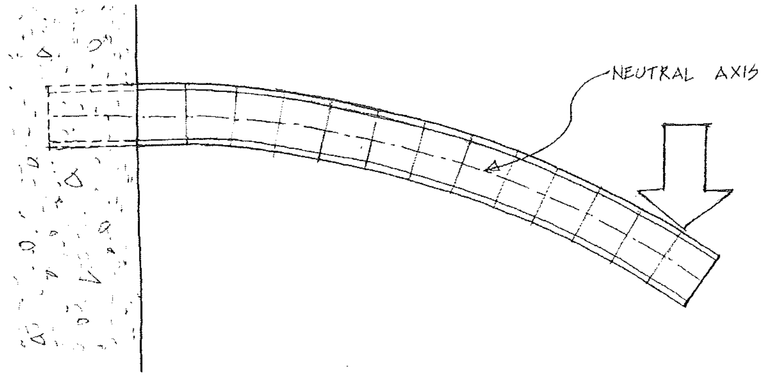
5.3 WIND REACTIONS OF ANCHORED COLUMNS



5.4 BUILDING BENT BY WIND AND CANTILEVER BENT BY LOAD

the structure are much larger than those due to axial, longitudinal loads. They are called *bending deflections* and are typical of basic structural elements, called *beams*, which are usually loaded at right angles to their longitudinal axis. A skyscraper under wind acts like a gigantic vertical beam stuck into the ground, much as a diving board acts in supporting the load of a diver (Fig. 5.4). Let us look in greater detail into the deformation of these *cantilevered beams*.

Consider a beam stuck into a wall and carrying a load at its tip—for example, a wooden jumping board with a man on the end. Under the man's weight, the beam deflects and its tip displaces downward. The beam bends, because the fibers of its upper part become longer and those of its lower part shorter. If one draws vertical lines on one side of the beam, these remain straight and perpendicular to the beam's upper and lower surfaces (Fig. 5.5). The lengthening and shortening of the upper and lower wood fibers are shown by the crowding of the vertical lines at the bottom and the opening-up of the lines at the top. The most remarkable feature of this deformation is that the fibers midway between the top and bottom maintain their original length. They are neither stretched nor shortened. Hence, they are neither under tension nor compression; they are unstressed. Moreover, we may also notice that the lengthening and shortening of the fibers in the upper and lower halves of the beam increase from being nonexistent at the middle fiber to the greatest values at the top and bottom of the beam, respectively. Since the board's behavior is linearly elastic, the tension and compression in the beam fibers grow linearly from zero at the middle to maximum values at the

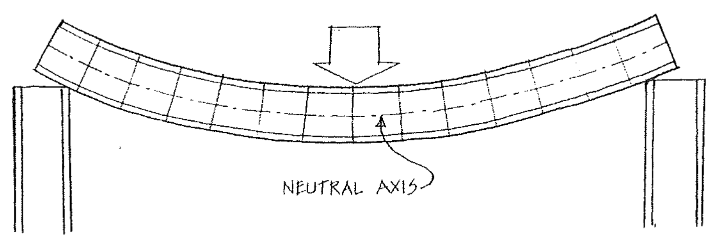


5.5 SECTIONS AND NEUTRAL AXIS IN BENT CANTILEVER

top and bottom fibers. This explains why the short lateral lines remain straight, as Leonardo first and then Navier knew (see Chapter 1).

In the compression of a column or the tension of a cable the loads are evenly divided between all the fibers of these elements. In a beam, instead, the extreme lower and upper fibers are highly stressed, while all the other fibers are stressed less and less as we approach the middle fibers. These middle fibers do not do any work while the others do more and more work the nearer they are to the top and bottom of the beam. Thus, most of the beam material is not utilized to its maximum capacity. A beam, whatever its material, is not a very efficient structural element as, on the average, its fibers work at half its allowable capacity.

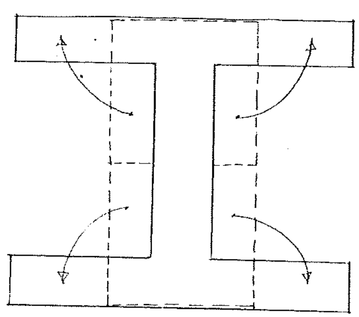
In a beam supported at its ends on columns, the crowding towards the top of the vertical lines on its side and their opening-up towards the bottom (Fig. 5.6) indicate, again, that the lower fibers are in tension and



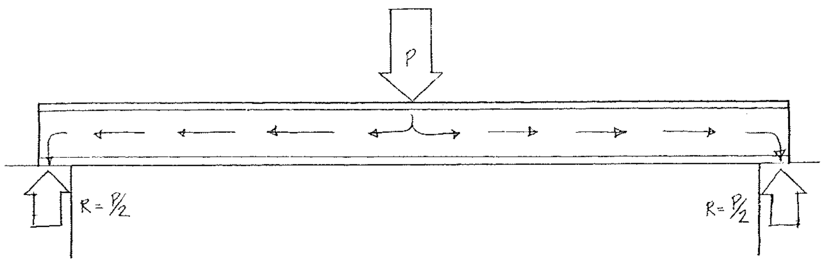
5.6 SECTIONS AND NEUTRAL AXIS IN END-SUPPORTED BEAM the upper in compression. This is why cracks appear at the bottom of beams made out of materials with a low tensile capacity, like stone or concrete, and why steel reinforcing bars are set at the bottom of concrete beams supported at their ends.

In channeling loads to the ground our most important and difficult task is to span horizontal distances so that loads may be carried across them. The structure of a bridge has no other purpose. Similarly, the floors of a building allow people to walk over the heads of people on lower floors. It is easy enough to support a load with a column right under it; it is much more difficult to carry the weight of a truck to the bridge supports so that the river may flow unimpeded below it. In buildings, distances (of up to 100 feet) are spanned by numerous horizontal beams. It is unfortunate that beams should be rather inefficient.

Is there anything one can do to improve the efficiency of beams? Realizing that all the material in the neighborhood of the middle fiber, which is called the *neutral axis*, is understressed, the thought occurs to move this material away from the neutral axis toward the top and bottom of the beam. If this is done as shown in Figure 5.7, the shape of the beam's transverse cut, or *cross-section*, becomes similar to that of a capital I. Of course, one cannot displace all the material near the middle fiber up to the top and down to the bottom of the beam. Some material must always connect the top and bottom parts of the I-beam, called its *flanges*, or they would become two separate, thin and flexible beams. This narrow vertical strip is called the beam's *web* and is typical of the steel beams used in the construction of high-rise steel buildings.



5.7 I-BEAM WITH SAME CROSS-SECTION AREA AS RECTANGULAR BEAM

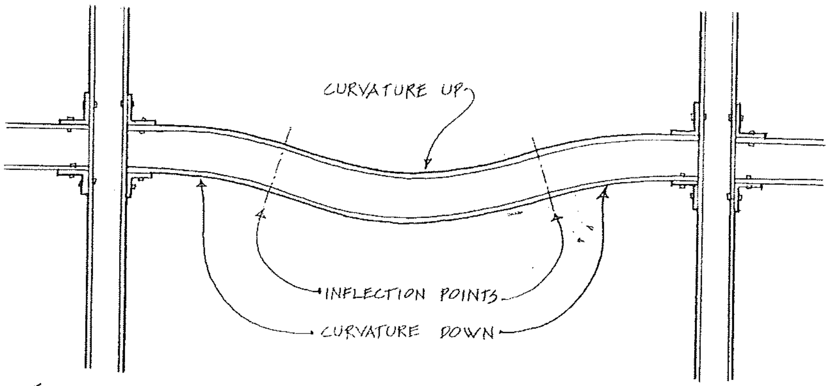


5.8 PATH OF LOAD ON AN END-SUPPORTED BEAM

I-beams of steel with wide flanges, called *wide flange sections*, are obtained by rolling heated and softened pieces of steel between the jaws of powerful presses and have flanges much wider than the top and bottom segments of a capital I. This is the most efficient shape a beam can be given to carry vertical loads horizontally from one point to another. One may think of a beam as a structural element that transfers vertical loads to the end supports along its horizontal fibers, as if the beam deflected the vertical flow of the loads by ninety degrees only to turn them around again in a vertical direction at the beam supports (Fig. 5.8).

Beams are made out of steel, aluminum, reinforced concrete, and wood. Their downward-bending displacements under load, larger than those of columns, must be limited lest they dip impossibly or the plaster begin to flake off the ceiling. Codes limit the bending deflection of a beam to less than the beam length divided by 360. The stiffness of a beam shape is increased by shifting part of the material away from its middle fibers and is measured by a quantity called the *moment of inertia* of the beam's cross-section, given in all beam manuals. Thus, deep beams are stiffer than shallow beams. On the other hand, the beam stiffness diminishes dramatically with increases in length; doubling the length of a beam makes it sixteen times more flexible.

If tension is characterized by lengthening and compression by shortening, *beam action* or *bending* is characterized by the curving of the beam. Whenever a straight element becomes curved under load, it develops beam action and the more curved it becomes, the larger its bending stresses. When a beam's ends curve up, as in a beam supported at its ends, its lower fibers are in tension and its upper fibers in compression. Whenever a beam curves down, like a cantilever supporting a balcony, the upper fibers are in tension and the lower in compression. (Hence, in



5.9 DEFLECTIONS IN A FIXED-END BEAM

a balcony beam of reinforced concrete, the reinforcing bars must be located towards the tensed top of the beam). In general, then, the tension appears on the opposite side of the curvature (see Figs. 5.5 and 5.6).

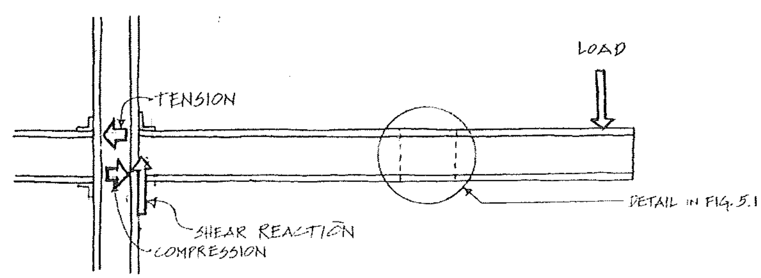
It has been tacitly assumed so far that a beam supported at the ends on two walls or two columns is not rigidly connected to the supports so that its ends rotate due to the bending deflections (see Fig. 5.6). Most of the time, instead, the beam is *fixed* into the supporting walls or rigidly connected to the top of the columns. In a reinforced concrete beam the simultaneous pouring of the concrete makes the columns monolithic with the beams, while in a steel structure beams are rigidly bolted to the columns. Such beams have ends which cannot rotate (or rotate only slightly). They are called *fixed-end* or *built-in* beams.

Fixed-end beams are particularly stiff and strong. They carry one-and-a-half times the load of a supported beam of the same length and they deflect five times less. Fixed-end beams deflect under load as shown in Figure 5.9. The figure shows that rigid connection to the supports makes the beam ends curve down, but that its middle curves up. This is why in the neighborhood of the supports of a fixed-end concrete beam the reinforcement is set near its top and in its middle portion near its bottom. There are two points in the beam at which the curvature changes from down to up. For a short length near these points the beam has no curvature and hence, develops no bending stresses. Such points are called *points of inflection*. Thus, the deflected beam shape allows a visualization of the distribution of bending stresses in the beam.

Lest it may be construed from this brief discussion of bending that the reinforcing bars in a concrete beam should always appear either near its top or its bottom, let us notice that a vertical column stuck into its foundation acts like a vertical cantilever under lateral loads and may be hit by the wind from either side. When the wind blows from the right, its fibers in tension are on the right and its compressed fibers on the left, but when the wind blows from the left, the role of the fibers is interchanged. In such columns reinforcing steel must appear near both sides of the column. In addition, the reinforcing bars serve to "stitch" the concrete and keep it together. To this purpose in most beams the bars, set at top and bottom, are connected by vertical hoops of steel, called *stirrups*, constituting a steel cage containing the concrete (see Fig. 5.12). The concrete, in turn, covers the steel bars, thereby preventing them from rusting. An insufficient concrete cover of the bars, which allows water to penetrate the beam and rust its reinforcement, has been known to make the reinforcement crumble inside the concrete until the structure collapses. This is a dangerous phenomenon because the rusting of the bars may not show on the outside of the structure to give warning of its condition.

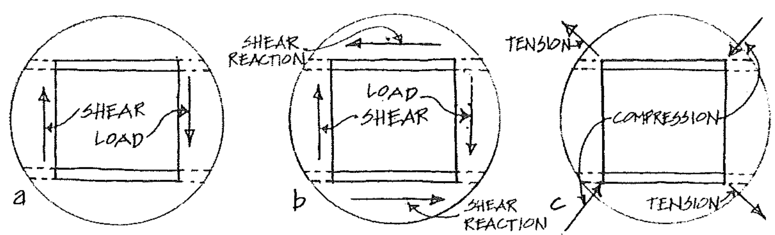
Shear

When a cantilever is loaded at its tip, it tends to rotate, but its rotational equilibrium is guaranteed by the action of the tensile forces in its upper fibers and the compressive forces in its lower fibers, which tends to make the beam rotate in the opposite direction (Fig. 5.10). Obviously its translational equilibrium must also be satisfied. This means that since the tip load acts down on it, an equal and opposite force or reaction must act up on the cantilever. This can only happen where

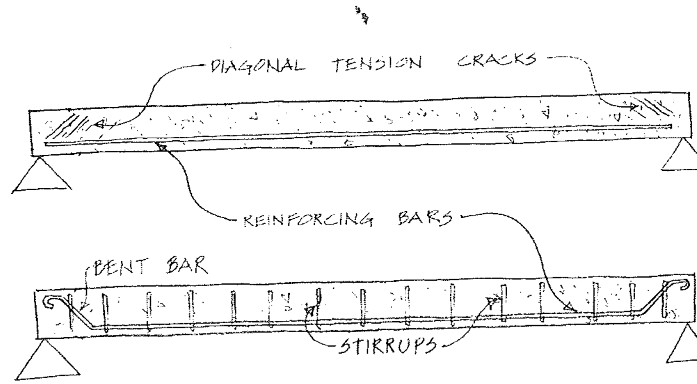


5.10 ROTATIONAL AND VERTICAL EQUILIBRIUM OF CANTILEVER the beam is supported. Hence, the support must exert an upward reaction equal to the load (Fig. 5.10), which is called *shear* because, together with the tip load, it exerts on the beam the type of action that shears exert on a steel sheet while cutting it.

Tension tends to move the particles of the material apart; compression pushes them together; shear makes them slide one with respect to the other. It may be rightly thought that shear action is a new and different type of structural action and that there are three *elementary* structural actions rather than two, as stated in Chapter 4. This is not so. That shear action is a combination of tension and compression is shown in Figures 5.11 a,b,c. Figure 5.11a shows a small cube cut out of a cantilever beam with two equal vertical and opposite forces acting on its left and right faces, the load down and the shear reaction up, which together guarantee the vertical equilibrium of the cube. But their opposite directions together with the distance between them tend to make the cube rotate clockwise, just as pulling with our right arm and pushing with our left tends to turn clockwise the wheel of our car. To guarantee the rotational equilibrium of the cube, equal and opposite forces, or shears, tending to rotate clockwise, just as pulling with our right arm and pushing with our left tends to turn clockwise the wheel of our car. To guarantee the rotational the particles of a beam vertically with respect to one another, it also necessarily tends to slide them horizontally. The equivalence of shear to a combination of tension and compression can now be seen in Figure 5.11c. This figure shows how the upper and left shears actually combine to become a tensile force directed up and to the left, while the lower and right shears combine to become an equal tensile force acting down and to the right. But the upper and right shears also combine to become a compression acting down and to the left, while the lower and left shears combine to become an equal compression up and to the right. It is thus



5.11 EQUIVALENCE OF SHEAR TO TENSION AND COMPRESSION AT RIGHT ANGLES



5.12 SHEAR TENSION-CRACKS AT BEAM SUPPORT

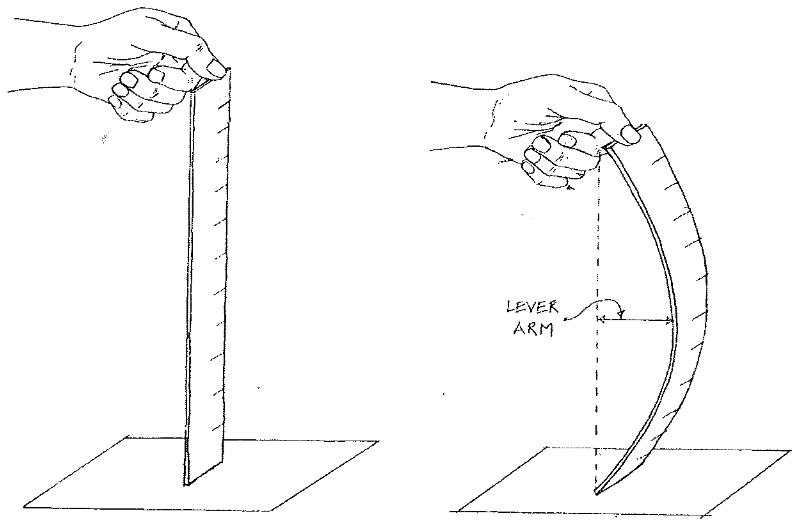
seen that shear is structurally equivalent to tension and compression at right angles to each other and at forty-five degrees to the shears. A physical proof of this equivalence is given by a reinforced concrete beam insufficiently reinforced against shear. Diagonal cracks at forty-five degrees appear near its support showing that the concrete tensile strength has been overcome by the tension component of the shear. Such cracks are avoided by bending the lower bars of the beam at an angle of forty-five degrees near the supports so as to absorb the tension due to shear, or by "stitching" the horizontal concrete layers with stirrups to prevent their sliding with respect to one another (Fig. 5.12).

Almost none of the problems presented by concrete beams exist in steel beams, since steel has as good a resistance to tension as to compression. And yet the next section will show that trouble can arise even in steel beams.

Buckling

It may seem strange that in a modern steel building the columns should have the same wide-flange shape as the beams, when this I-shape was shown to be ideal for bending while columns are not bent, but compressed. But this choice eliminates one of the most dangerous structural phenomena called *buckling:* the bending of a straight element under compression.

If one pushes down with increasing pressure on a vertical thin steel ruler supported on a table (Fig. 5.13), the ruler at first remains straight,



5.13 BUCKLING OF THIN RULER UNDER AXIAL COMPRESSION

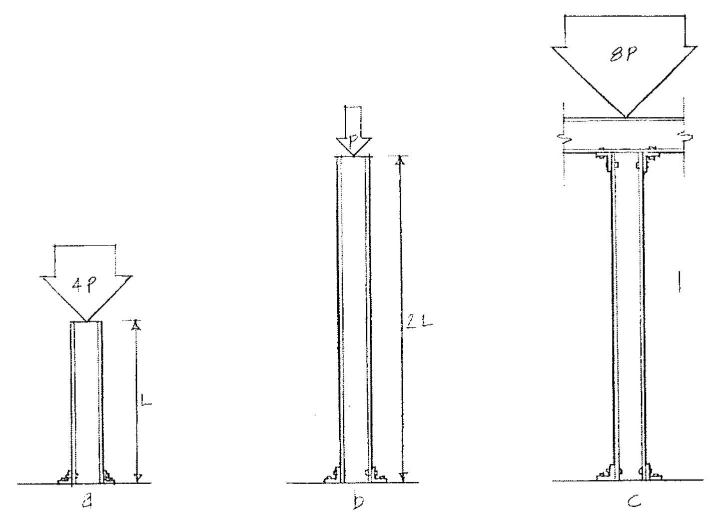
but there comes a point when, rather suddenly, the ruler bends out. It can be proved that a *perfectly* straight ruler acted on by a compressive force *perfectly* aligned with its axis will bend out suddenly at a given value of this force, called its *critical value*. (The real ruler does not bend so suddenly because it is never perfectly straight.) The moment the ruler bends out, the compressive force acquires a lever arm with respect to its axis (Fig. 5.13) and bends it progressively more. This is a chain reaction where the more the ruler bends, the larger the lever arm becomes. This increases the bending action of the force, which increases the lever arm, and so on. Very soon the ruler fails in *bending*. The column is said to become *unstable* when the load reaches its critical value.

This behavior is typical not only of thin columns, but of any thin element under compression and has acquired great significance due to the thin sections allowed by modern, strong materials. The columns of a Greek temple could never buckle because they were chunky and short. The slender columns of a modern building are much more likely to buckle. Since buckling is a phenomenon involving bending, it becomes clear why modern steel columns have the shape of wide-flange beams.

Their resistance to buckling is magnified by this shape without a costly increase in material.

The load capable of buckling a column, or its *critical load,* depends on the slenderness, the material, and the way a column is supported. Since the moment of inertia was found to be a quantity characterizing the amount of material moved away from the neutral axis of a beam and hence measuring its bending stiffness, it is not surprising that the buckling load increases in proportion with the moment of inertia of the wide-flange columns. The longer a column, the slenderer it becomes and its buckling load is reduced in proportion to the square of its length. A column twice as long as another has a buckling load four times smaller. A column stuck into its foundation and free to move at its top (a cantilevered column) has a buckling load eight times smaller than the same column stuck into its foundation and rigidly connected to a floor at its top (Fig. 5.14). Finally, the stiffer the column's material, the stronger the column. A steel column is three times stronger against buckling than an identical aluminum column.

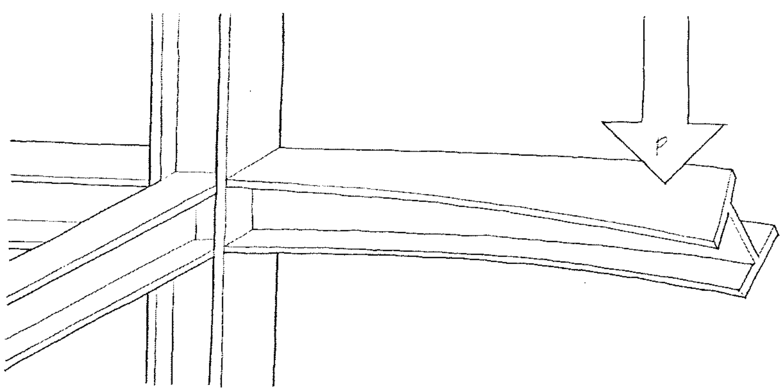
It is interesting to notice that buckling is a consequence of the basic "least work law" of nature. If an increasing load is applied to the top of



5.14 BUCKLING LOADS OF (a AND b) CANTILEVERED AND (c) FIXED-END COLUMNS a column, the load first comes down by compressing and shortening the column. Since loads always tend to settle in their lowest balanced position, this type of lowering of the load would go on indefinitely, were it not for the fact that at a certain value of the load, the column can lower it further in two ways: either by continuing to shorten in compression or by bending out. As soon as the work required to further lower the load is less in bending than in compression, the column follows the easier path and bends. The column buckles.

The shape given modern steel columns may prevent the buckling of the column as a whole, but may still allow the buckling of some of its parts when these are thin. Thin webs are most sensitive to local buckling and, at times, must be stiffened to prevent it. Buckling of the web of steel beams may occur near the suports due to the compressive component of the shear. The lower flange of a cantilevered wide-flange beam, being thin and compressed in bending by vertical loads, may also buckle. This buckling occurs in a lateral direction, as shown in Figure 5.15, and twists the beam, besides bending it.

Buckling is one of the main causes of structural failure. The roof of the Hartford Civic Center hockey rink was a *space frame* (described in Chapter 9) made out of steel bars. It covered an area of 360 feet by 300 feet and was supported on four massive pillars of concrete set 40 feet in from its corners. Its upper bars were compressed by its enormous dead load of 1,400 tons and by the weight of snow, ice, and water accumulated on it. After standing for four years, it suddenly failed, at four o'clock in the morning following a heavy snowstorm in 1978. Fourteen hundred tons of steel came crashing down on the floor and stands of the hockey rink, where only five hours earlier 5,000 spectators sat watching a game. The roof collapsed in less than ten seconds. This structural failure is attributed by engineers entrusted with its investigation by the city of Hartford to the buckling, at first, of only a few compressed bars, which shifted to adjoining bars the load they were supposed to support. The overloaded adjoining bars in turn buckled and the progressive spreading of buckling to more and more bars produced the collapse of the entire roof. This dramatic occurrence shows that one of the most dangerous characteristics of a buckling failure is its suddenness, which gives no warning. Whenever a structure under load chooses the easy path of bending rather than the foreseen path of compression, the structure may fail. Good engineering judgment, correctly shaped and supported elements, strong materials, and careful supervision during construction are needed to avoid this particularly tricky and sensitive structural behavior.



5.15 LATERAL BUCKLING OF WIDE-FLANGE BEAM